Overview

All power supplies require some type of feedback loop to regulate the output voltage. A simplified diagram of a buck controller is shown in Figure 1 with a pulse width modulator (PWM) and an error amplifier / compensator.

For this feedback loop to be stable, negative feedback is used. This means that the output voltage (V_{OUT}) is compared to the reference voltage (V_R) and the difference (V_R - V_{OUT}) is amplified by the error amplifier. The output of the error amplifier is called the error voltage. The error voltage polarity is inverted with respect to V_{OUT} and is therefore -180° out of phase with V_{OUT}. The goal of the feedback loop is to minimize the error between V_{OUT} and V_R. The error is small when the overall feedback loop gain is high. Ideally, a feedback loop would have infinite gain for all frequencies, thus providing perfect regulation regardless of the demands of the load.

\[ f_n = \frac{1}{2\pi \sqrt{LC}} \quad \text{Eq. [1]} \]

Also, the components in the LC filter are not ideal because they contain parasitic impedances. In many capacitors used in output filters, the Equivalent Series Resistance (ESR, shown as R_C in Figure 1) is large enough to alter the response of the LC filter at frequencies that impact the control loop design. This is called the ESR zero (f_{zer}). It has the effect of changing the LC filter's attenuation slope from -40dB (-2 slope) per decade to -20dB (-1 slope) per decade and adding 90° of phase back into the feedback loop. The ESR zero of the output capacitor is calculated using Eq. [2]:

\[ f_{zer} = \frac{1}{2\pi CR_C} \quad \text{Eq. [2]} \]
Closing the Control Loop

Frequency compensation is used to achieve the goal of less than $360^\circ$ of phase shift when the open-loop gain is above 0dB. In the circuit of Figure 1, compensation is provided by the type 3 amplifier. The easiest method of compensating control loops using Digital-DC Technology is by using compensation pin-strap settings listed in the datasheet for each device. Zilker Labs, Inc. also offers user-friendly tools for calculating compensation coefficients using regulator design constraints and filter component parameters.

LC filters exhibit resonant behavior. Near the natural frequency ($f_n$), the LC filter transfer function will effectively amplify the input signal. The extent to which this occurs is determined by the quality factor ($Q$). In effect, the $Q$ of the circuit is a measure of the gain the LC filter exhibits at $f_n$ and how abrupt the phase shift from 0 to -180° will be. In power supply applications, it is also indirectly a measure of the circuit’s efficiency: the lower the resistances in the circuit, the higher the $Q$. However, a high $Q$ also corresponds to an abrupt phase shift from 0 to -180°.

For a simplified power stage and output filter consisting of high-side MOSFET on-resistance $R_{ON-H}$, low-side MOSFET on-resistance $R_{ON-L}$, output inductor $L$ having resistance $R_L$, an output capacitance array of $n$ parallel capacitors having capacitance $C$ and equivalent series resistance $R_e$ and for a load resistance $R_O$, the natural frequency and quality factor of the circuit are given by Equations [3] and [4], respectively.

$$f_n = \frac{1}{2\pi \sqrt{nC \left( \frac{R_e}{n} + R_O \right) \left( \frac{L}{R_e + R_O} \right)}}$$

or

$$f_n = \frac{1}{2\pi \sqrt{LC \left( \frac{R_c}{n} + nR_O \right) \left( \frac{R_e + R_O}{R_e + R_O} \right)}}$$

Eq. [3]

$$Q = \frac{L}{R_e + R_O} + nC \left( \frac{R_c}{n} + R_O \right)$$

or

$$Q = \frac{\sqrt{LC \left( \frac{R_c}{n} + nR_O \right) \left( \frac{R_e + R_O}{R_e + R_O} \right)}}{L + C \left( \frac{R_c}{n} + R_O \right) + nR_O R_c}$$

Eq. [4]

where $R_e$ is the effective resistance of the MOSFETs averaged over a switching period combined with the inductor resistance. For a given duty cycle, $D$, $R_e$ is given by

$$R_e = D \cdot R_{ON-H} + (1 - D) \cdot R_{ON-L} + R_L$$

Eq. [5]

For $Q > 1$, which is the case for many low-cost power supplies, the phase can be assumed to shift from 0 to -180° at $f_n$.

The PWM has a fixed gain of

$$G_{PWM} = \frac{V_{IN}}{V_{SAW}}$$

Eq. [6]

where $V_{SAW}$ is the peak amplitude of the PWM sawtooth waveform used in the PWM controller. $G_{PWM}$ is a constant and has no phase impact on the feedback loop. The power stage gain, $g_{PS}$ (i.e., control-to-output transfer function) consists of a DC gain, $G_{PS}$, and a frequency-dependent component which is a function of $f_{zer}$, $f_n$ and $Q$.

$$g_{PS} = G_{PS} \frac{1 + \frac{s}{2\pi f_{zer}}}{1 + \frac{s}{2\pi f_{zer}} + \frac{s^2}{(2\pi f_{zer})^2}}$$

Eq. [7]

This transfer function is plotted in Figure 2. We define a term, $G_{FIX}$, which combines all the DC gains in the loop except that of the compensator.

$$G_{FIX} = G_{PWM} G_{PS}$$

Eq. [8]

If $g_{COMP}$ represents the gain of the compensator, the open-loop transfer function of the simplified buck converter is then
The desired feedback open-loop response is shown in Figure 3.

The goal of the compensation circuit is to modify the gain and add phase to the frequency response of the modulator and LC filter sections of the power supply (Figure 2) to achieve the desired feedback gain and phase (Figure 3). Comparing Figure 2 and Figure 3 reveals what the frequency response of the compensation circuit should be. The compensation circuit frequency response is shown in Figure 4.

In a voltage mode analog power supply, this compensation is done with resistors and capacitors configured to provide poles, zeroes, and gain in the error amplifier.

To provide the highest possible gain at low frequencies, the compensator is generally chosen to be an integrator. This means that the compensator has a pole at the origin (DC), a -1 gain slope and a phase shift of -90°. Because the phase shift due to the error amplifier is -180°, the phase shift due to the LC filter is -180° and the phase shift of the integrator is -90°, the compensator must provide a positive phase boost. In addition, because the LC filter has a -2 gain slope and the integrator has a -1 slope, the compensator must have a +2 gain slope to yield the desired net gain slope of -1. The compensator accomplishes this with two zeroes placed near the LC filter natural frequency, $f_n$.

These zeroes provide the +180° phase shift and +2 slope needed to counteract the integrator and LC filter characteristics.
Although the loop gain is usually below 0dB before the ESR zero, the compensator often provides a pole (-90°, -1 slope) to neutralize the effects of the ESR zero (+90°, +1 slope). Finally, due to limitations of real-world amplifiers, the compensator provides an additional pole to keep the error amplifier from operating beyond its gain-bandwidth product limit and to keep the switching frequency ripple from interfering with the operation of the PWM circuitry.

To complete the compensation circuit design, the amount of gain required by the compensation circuit must be determined. First, select a cross-over frequency ($f_{xo}$), which is a fraction of the switching frequency ($f_{sw}$). Generally, a ratio of 1/20 of the switching frequency is a conservative starting point (≈ $f_{sw}/2\pi$ is the theoretical max[1]). The gain needed from the compensator is determined by assuming the control loop is compensated so the gain plot crosses 0dB at a -1 slope as shown in Figure 3. The DC gain needed for this analog compensator ($G_{ACMP-DC}$) is calculated by assuming a -1 slope for the compensated control loop for a given crossover frequency. This can be expressed as

$$G_{ACMP-DC} = \frac{f_{xo}}{f_{sw}G_{FIX}} = \frac{1}{20G_{FIX}} \quad \text{Eq. [10]}$$

As noted above, the compensator has two zeroes at the LC resonant frequency ($f_n$). A pole is added at the ESR zero ($f_{esr}$) to keep the gain of the open-loop transfer function at a slope of -1, and an additional pole is added at $f_{sw}/2$ to reject noise. The amplifier circuit shown in Figure 5 is used to achieve the compensation.[2]

The component values are calculated using the following equations:

$$R_2 / R_1 = G_{ACMP-DC} \quad C_1 = 1/(2\pi f_n R_2)$$
$$C_2 = 1/(2\pi f_{esr} R_3) \quad C_3 = 1/(2\pi f_n R_1)$$
$$R_3 = 1/(2\pi (f_{sw}/2)C_3)$$

The combination of the buck converter and compensation amplifier transfer functions yields the desired open-loop response shown in Figure 3.

Once the compensation values have been determined, the stability of the power supply can be evaluated. Using the equation for the buck converter transfer function and the gain, pole and zero locations from the compensation circuit, the resulting cross-over frequency, phase and gain margin can be evaluated. Although, ideally, the phase margin using this technique would be 90° and the gain margin would be 20dB or more, in reality 60° and 6dB are considered to be good design goals that strike a balance between performance and stability. Decreasing phase margin to less than 60° or gain margin to less than 6dB will increase the output voltage overshoot with transient output loads.

Digital PWM

The block diagram for a power supply controlled by a digital PWM is shown in Figure 6. The digitally controlled power supply differs from the analog version only by the values shown in the PWM block and the compensation block. The gain of the digital PWM block is seen in the functional block diagram of Figure 7.

The PWM block is derived from several functions within the device. These functions, although implemented in high speed mixed signal circuitry, can be modeled with constant values because of the relatively high frequency at which these blocks operate compared to the frequencies of interest to the control loop (i.e., >10X the switching frequency).
The output voltage is first applied to a programmable gain amplifier (PGA) that has a gain of between 0.2 and 2.0. This amplifier is used to enable higher accuracy in the setting of the output voltage. Its gain is cancelled by the gain corrector, which modifies the compensation settings to yield a net gain of 1. The gain correction function also compensates for changes in the input voltage by the same process. The gain corrector normalizes the input voltage to 5V and automatically modifies the gain so that the net result is a loop gain that would exist if the input were at 5V.

Following the PGA, the error signal is applied to the A/D converter, which converts this error voltage into a digital value. Its gain is \(1/V_{step}\) (1/5 mV) or 200.

Following the A/D converter is the anti-aliasing filter, which is a multi-stage filter that provides signal gain and high frequency noise rejection. It eliminates the need for the second pole used in the analog compensator. As can be seen in Figure 8, it has a fixed gain of 64 at low frequencies, but a large attenuation at the switching frequency without adding excessive phase loss. This phase loss can be expressed as a function of the switching frequency and is discussed later as part of the compensation.

\[
G_{\text{FIX}} = \frac{5 \cdot 200 \cdot 64}{2^{18}} G_{PS} \quad \text{Eq. [11]}
\]

The transfer function of the digital voltage mode converter can now be expressed as an equation in the same form as the analog controller:

\[
g = g_{\text{COMP}} G_{\text{FIX}} \left( \frac{1 + \frac{s}{2\pi f_{\text{zer}}} + \frac{s^2}{2\pi f_{n}Q} + \frac{s^2}{(2\pi f_{n})^2}}{1 + \frac{s}{2\pi f_{n}Q} + \frac{s^2}{(2\pi f_{n})^2}} \right) \quad \text{Eq. [12]}
\]
As mentioned earlier, the analog compensator of Figure 5 provides two zeroes to cancel the two poles of Equation [12] (at $f_n$), but it also provides three poles. One of these poles cancels the ESR zero of Equation [12] (at $f_{zesr}$), a high frequency pole (at $f_{sw}/2$) improves noise immunity and the third pole is at the origin – an integrator which provides the desired response of a slope of -1. Finally, the analog compensator provides a DC gain term ($G_{ACMP-DC}$) which ultimately determines the crossover frequency and impacts phase margin and gain margin as well. To obtain the ideal response of Figure 3 and provide noise rejection above $f_{sw}/2$, the analog compensator’s transfer function must be

$$g_{ACMP} = G_{ACMP-DC} \left( \frac{1 + \frac{s}{2 \pi f_n} \left( \frac{s^2}{2 \pi f_{zesr}^2} \right)}{1 + \frac{s}{2 \pi f_{sw}}} \right)$$ \text{ Eq. [13]}$$

Transforming Equation [12] to the digital domain yields an equation of the form

$$g_z = g_{ZCOMP} G_{ZFIX} \left( 1 - dz^{-1} \right)$$ \text{ Eq. [14]}$$

where $g_{ZCOMP}$ is the $z$-domain compensator, $G_{ZFIX}$ is a transcendental function of $G_{FIX}, f_n, f_{zesr}$ and $f_{sw}$ representing the DC gain in the $z$-domain, $d$ is a transcendental function of $f_{zesr}$ and $f_{sw}$, and $b$ and $c$ are transcendental functions of $f_n$ and $f_{sw}$. It should be noted that, for $Q > 0.5$, $b$ becomes complex but may be converted to a real number using Euler’s identity.

The digital control loop is compensated in much the same way as the analog control loop. However, because the digital control loop has an anti-aliasing filter that provides noise rejection above $f_{sw}/10$, the poles at $f_{zesr}$ and $f_{sw}/2$ in Eq. [13] are not required in the digital compensation filter. For this reason, the crossover frequency should occur at least one octave before the ESR zero ($f_{xo} \leq f_{zesr}/2$). The pole at the origin is implemented using an accumulator. Again, a crossover frequency of $f_{xo} - f_{sw}/20$ is selected and the DC gain of the digital compensator is calculated, using an equation similar to Equation [10] for the analog compensator:

$$G_{ZCMP-DC} = \frac{f_{xo}}{f_{sw} G_{ZFIX}} = \frac{1}{20 \cdot G_{ZFIX}}$$ \text{ Eq. [15]}$$

Like the analog compensator, the digital compensator has two zeroes at the LC resonant frequency ($f_n$). Instead of an op-amp circuit, the digital compensator uses a recursive filter, which is simply an adder with four inputs: 1) the output of the A/D converter (the error voltage) multiplied by a constant, 2) the error voltage from the previous switching cycle (shown as $z^{-1}$) multiplied by another constant, 3) the integrator and 4) the error voltage from the switching cycle two cycles previous (shown as an additional $z^{-1}$) multiplied by a third constant. This filter is shown schematically in Figure 9.

Figure 9. Digital Compensator / Recursive Filter

By substituting $z^{-1} = e^{-sT}$ for each delay (where $T$ is the sampling period), the digital compensator / recursive filter can be represented as follows:

$$g_{ZCOMP} = \frac{A + Bz^{-1} + Cz^{-2}}{(1 - z^{-1})}$$ \text{ Eq. [16]}$$

or

$$g_{COMP} = \frac{A + Be^{-sT} + Ce^{-2sT}}{(1 - e^{-sT})}$$

The term $1/(1-e^{-sT})$ is due to the accumulator and models both the gain and phase response of the integrator. Although in transcendental form, the digital compensator / recursive filter can be configured to have the same response as the analog compensator by selecting appropriate constants for $A, B$ and $C$ in Eq. [16].
To do so, set like terms equal in the numerator of Eq. [16] and the denominator of Eq. [14], being sure to divide by the compensator DC gain, $G_{ZCMP-DC}$. This yields Equations [17] for real zeroes (at $f_{z1}$ and $f_{z2}$) and Equations [18] for complex zeroes (at $f_n$ for $Q > 0.5$). To simplify these equations, new terms $r_1$ and $r_2$ are defined for Eq. [17] and $r$ and $\theta$ are defined for Eq. [18].

Compensation Coefficients for Real Zeroes:

\[
\begin{align*}
&\text{Let} \quad r_1 = e^{-2\pi f_{z1}/f_{sw}}, \quad r_2 = e^{-2\pi f_{z2}/f_{sw}} \\
&A = \frac{G_{ZCMP-DC}}{(1 - r_1) \cdot (1 - r_2)} \quad \text{Eq. [17]} \\
&B = -A \cdot (r_1 + r_2) \\
&C = A \cdot r_1 \cdot r_2
\end{align*}
\]

Compensation Coefficients for Complex Zeroes:

\[
\begin{align*}
&\text{Let} \quad r = e^{\pi f_n/f_{sw}}, \quad \theta = \frac{2\pi f_n}{f_{sw}} \cdot \sqrt{1 - \frac{1}{4 \cdot Q^2}} \\
&A = \frac{G_{ZCMP-DC}}{1 - 2 \cdot r \cdot \cos \theta + r^2} \quad \text{Eq. [18]} \\
&B = -A \cdot 2 \cdot r \cdot \cos \theta \\
&C = A \cdot r^2
\end{align*}
\]

As with the analog compensator, select the zero frequencies and the gain desired for the compensator stage. Solve the equations to yield the appropriate $A$, $B$, and $C$ coefficients. These coefficients are loaded into the digital compensator using a PMBus command. Additional details on the use of the PMBus for loading compensation coefficients are in Application Note AN2013.[3] The power supply response can be simulated and tested in the same manner as an analog controller.

The digital compensator also provides a means of correcting for the $Q$ of the LC filter. Using the calculated coefficients, both the gain and phase of the LC filter are cancelled by the digital compensator, as shown in Figure 10.

\[
I_{95\%} \approx 3 \cdot \tau_{xo} = \frac{3}{2\pi f_{xo}} \quad \text{Eq. [19]}
\]
The response to the step load may have some overshoot. However, more than three oscillations indicate marginal stability; an exponential decay is preferred.

The control-loop response should also be verified by injecting a frequency swept signal into the loop and measuring the frequency response with a network analyzer. This is done by placing a small resistor (10 – 100Ω) between the output and the VSEN pin and injecting the signal into the feedback loop by applying a floating AC signal across this resistor, as shown in Figure 11. The voltage from VSEN to ground is the input signal (sometimes called reference signal), and the power supply output voltage \(V_{OUT}\) is the output signal (sometimes called the test signal). The network analyzer will read out the relative phase and gain of the transfer function: output signal/input signal. The goal is to verify a phase margin of >60° and a gain margin of >6dB over a range of operating conditions. The test signal should be monitored with an oscilloscope and adjusted so that the test signal peak amplitude does not exceed 1% of the \(V_{OUT}\). Excessive test signal amplitude will result in erroneous loop gain measurements. Proper correlation between measurements and simulations depends on the accuracy of the models used, and Zilker Labs, Inc. is constantly working to ensure the accuracy of its models.

**References**


**Revision History**

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<tr>
<td>09/27/2006</td>
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![Figure 11. Loop Gain Measurement](image-url)